

3-10 R-module N: fin. rep's

$$\Leftrightarrow \exists m, n \in \mathbb{N} \text{ s.t. } R^n \rightarrow R^m \rightarrow N \rightarrow 0 \text{ exact}$$

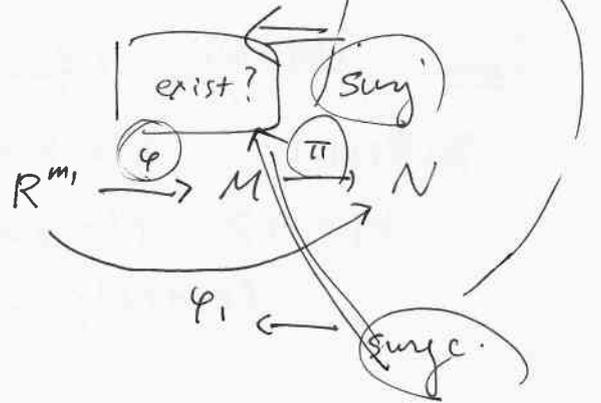
$$0 \rightarrow L \xrightarrow{\iota} M \xrightarrow{\pi} N \rightarrow 0 \text{ exact.}$$

L, N: fin. rep. R-module \rightarrow M: fin. rep.

$$\begin{array}{c} R^{n_1} \xrightarrow{\psi_1} R^{m_1} \xrightarrow{\varphi_1} N \rightarrow 0 \\ R^{n_2} \xrightarrow{\psi_2} R^{m_2} \xrightarrow{\varphi_2} L \rightarrow 0 \end{array}$$

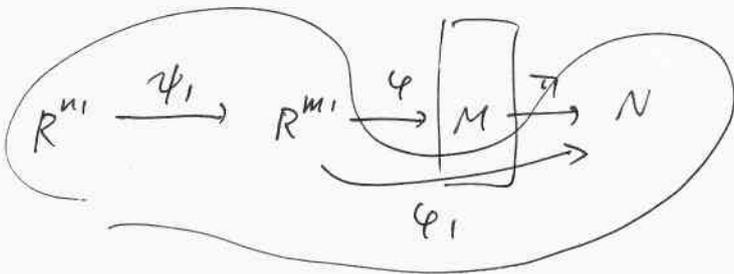
R-hom. $\varphi: R^{m_1} \rightarrow M$

$$\varphi_1 = \pi \circ \varphi$$



$$\text{Im}(\varphi \circ \psi_1) \subset \text{Ker } \pi = \text{Im } \iota$$

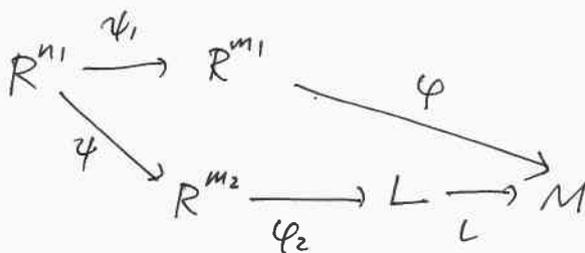
$R^{n_1} \rightarrow M$ exact 故.



exact 故 $\text{Im}(\varphi_1 \circ \psi_1) = 0$

$\therefore \text{Im}(\varphi \circ \psi_1) \subset \text{Ker } \pi$ (2つ...と合成の 行先が0) 故

$$\Rightarrow \exists \psi: R^{n_1} \rightarrow R^{m_2} \text{ s.t. } \iota \circ \varphi_2 \circ \psi = \varphi \circ \psi_1$$



$$\therefore R^{n_1} \times R^{n_2} \xrightarrow{g} R^{m_1} \times R^{m_2} \xrightarrow{f} M \rightarrow 0$$

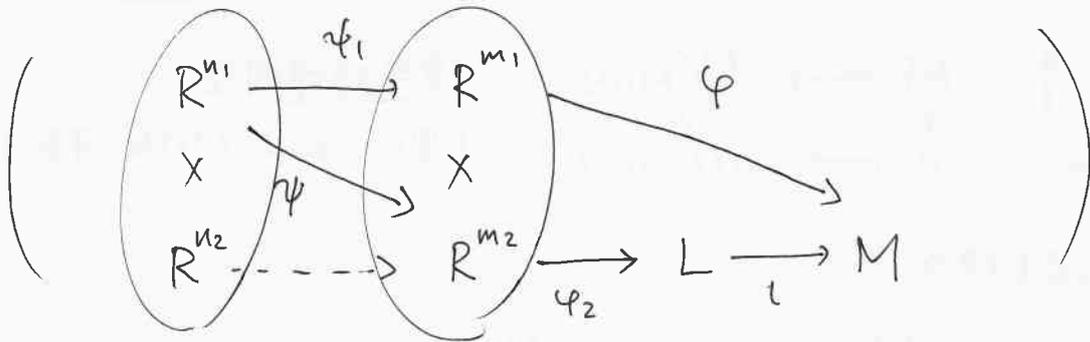
$$(x, y) \mapsto (\psi_1(x), \psi(x) - \psi_2(y))$$

$$(x, y) \mapsto \varphi(x) - \iota \circ \varphi_2(y)$$

$$\text{Im } g = \text{Ker } f$$

$$\Rightarrow R^{n_1+n_2} \rightarrow R^{m_1+m_2} \rightarrow M \rightarrow 0 : \underline{\text{exact.}}$$

~~~~~ (R.S. remark)



exist  $\psi \because \text{Im}(\varphi \circ \psi_1) \subset \text{Ker } \pi = \text{Im } \iota$

~~~~~  $\exists \iota \in L$  exists.

~~~~~  $R^{m_2} \xrightarrow{\varphi_2} L \rightarrow 0$  : exact  $\rightarrow \varphi_2$  surject.

i.e.  $\exists y_2 \in R^{m_2}$

$\therefore R^{n_1} \xrightarrow{\psi} R^{m_2}$  cohoresp.

exist R-hom.  $\varphi$  and  $\psi$ .

